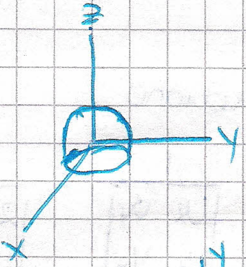
PARCIALES VARIAS VARIABLES

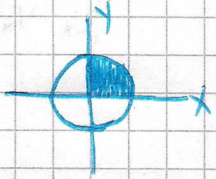
1. ⓐ $z = 1 - x^2 - y^2$ el volumen se puede Expresar

plano xy
primer octante



dz dy dx
dx dz dy

$z = 1 - y^2$

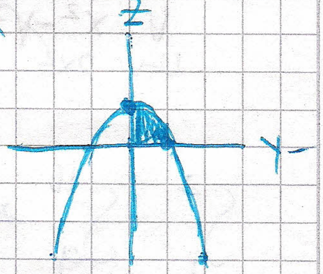


$z = 1 - x^2 - y^2$

$y^2 = \sqrt{1 - x^2 - z}$

$y = \sqrt{1 - x^2}$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} dz\,dy\,dx$$

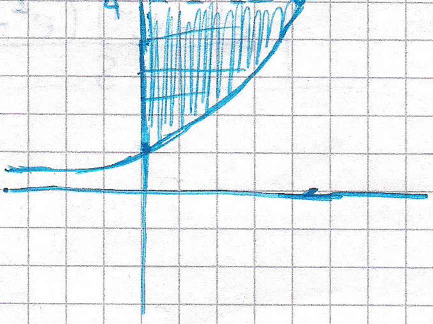


$$\int_0^1 \int_0^{1-y^2} \int_0^{1-z-x^2} dx\,dz\,dy$$

ⓑ.

$$\int_1^4 \int_0^{\ln(y)} f(x,y)\,dx\,dy$$

$x = \ln(y)$
$x = 0$
$y = 1$
$y = 4$



dy dx ?

$e^x = y$

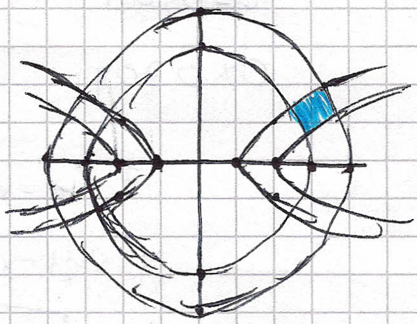
$$\int_0^{e^4} \int_{e^x}^4 f(x,y)\,dy$$

DD MM AA

② $\iint_D xye^{2x^2}\,dA$

$x^2+y^2=9$
$x^2+y^2=16$

$x^2-y^2=1$
$x^2-y^2=2$



Jacobiano

$$\frac{1}{\begin{vmatrix} U_x & U_y \\ V_x & V_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2x & 2y \\ 2x & -2y \end{vmatrix}}$$

$$= \frac{1}{-4xy-4xy}$$

$$= \left|\frac{1}{-8xy}\right|$$

$$= \frac{1}{8xy}$$

$U = x^2 \cancel{+y^2}$
$V = x^2 \cancel{-y^2}$
$U+V = 2x^2$

$$\int_1^2 \int_9^{16} \cancel{xy}\,e^{2x^2}\frac{1}{8\cancel{xy}}\,du\,dv = \int_1^2\int_9^{16} \frac{e^{2x^2}}{8}\,du\,dv$$

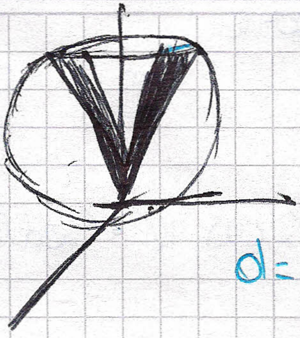
$$= \int_1^2\int_9^{16} \frac{e^u \cdot e^v}{8}\,du\,dv$$

$$\boxed{\frac{(e^{16}-e^9)(e^2-e)}{8}}$$

③ $z = \sqrt{3(x^2+y^2)}$

$z = \sqrt{x^2+y^2}$

$x^2+y^2+z^2 = 2z$



$M = ?$

$d = \frac{1}{z^2+y^2+x^2}$

Esfericos

$\cancel{\rho}\cos\varphi = \sqrt{3}\,\cancel{\rho}\,\text{sen}\,\varphi$

$\frac{1}{\sqrt{3}} = \frac{\text{sen}\,\varphi}{\cos\varphi}$

$\text{Tan}\,\varphi = \frac{1}{\sqrt{3}}$

$\varphi = \frac{\pi}{6}$

$\rho\cos\varphi = \rho\,\text{sen}\,\varphi$

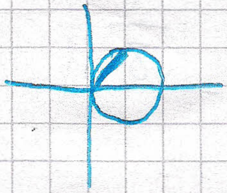
$\text{Tan}\,\varphi = 1$

$\varphi = \frac{\pi}{4}$

$\frac{\pi}{6} \leq \varphi \leq \frac{\pi}{4}$

$\rho^2 = 2\rho\cos\theta$

$\rho^2 = 2\rho\cos\varphi$



$\rho = 2\cos\varphi$

$$\int_0^{\pi/2}\int_{\pi/6}^{\pi/4}\int_0^{2\cos\varphi} \frac{1}{\rho^2}\,\rho^2\,\text{sen}\,\varphi\; d\rho\, d\theta\, d\varphi$$

$$\int_0^{\pi/2}\int_{\pi/6}^{\pi/4}\int_0^{2\cos\varphi} \text{sen}\,\varphi \; d\rho \, d\varphi \, d\theta$$

$$\int_0^{\pi/2}\int_{\pi/6}^{\pi/4} 2\cos\varphi \cdot \text{sen}\,\varphi \; d\varphi \, d\theta$$

$u = \text{sen}\,\varphi$

$du = \cos\varphi \, d\varphi$

$$2\int_0^{\pi/2}\int_{\pi/6}^{\pi/4} u \, du \, d\theta$$

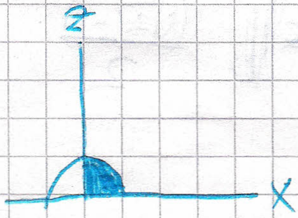
$$2\int_0^{\pi/2}\int_{1/2}^{\sqrt{2}/2} u \, du \, d\theta$$

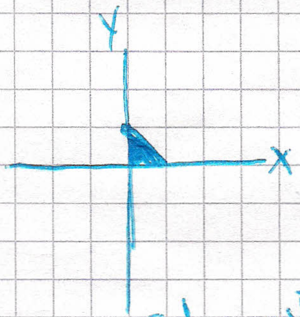
$$2\int_0^{\pi/2} \frac{\frac{2}{4}}{2} - \frac{\frac{1}{4}}{2} \qquad \frac{\frac{1}{4}}{2} \qquad \frac{1}{8}$$

$$\frac{1}{4}\,\frac{\pi}{2} \qquad \boxed{\frac{\pi}{8}}$$

④ $\int_0^1 \int_0^{1-x} \int_0^{1-x^2} f(x,y,z)\ dz\,dy\,dx$

dy dx dz ?





$z=0$
$z=1-x^2$

$y=0$
$y=1-x$

$x=0$
$x=1$

$x^2=1-z$
$x=\sqrt{1-z}$

$$\int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} dy\,dx\,dz$$

⑤ Sea D

$0 \leq x \leq \pi/4 \quad sen(x) \leq y \leq cos(x)$

$1/3 \leq f(x,y) \leq 1/2$

$$\int_0^{\pi/4} \int_{sen(x)}^{cos(x)} dy\,dx$$

$$\int_0^{\pi/4} cos(x) - sen(x)\,dx$$

[illegible] $sen(x) + cos(x) \Big|_0^{\pi/4} dx$

$(\sqrt{2} - 1) = dA$

$$\int_0^{\pi/4}\int_{sen(x)}^{cos(x)} \frac{1}{3}\, dA \le \int_0^{\pi/4}\int_{sen(x)}^{cos(x)} f(x,y)\,dA < \int_0^{\pi/4}\int_{senx}^{cos(x)} \frac{1}{2}\, dA$$

$$\frac{\sqrt{2}-1}{3} \le \int_0^{\pi/4}\int_{sen(x)}^{cos(x)} f(x,y)\,dA \le \frac{(\sqrt{2}-1)}{2}$$

⑥ $z=x^2$

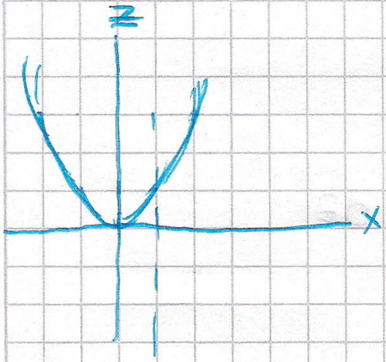
$x=1$

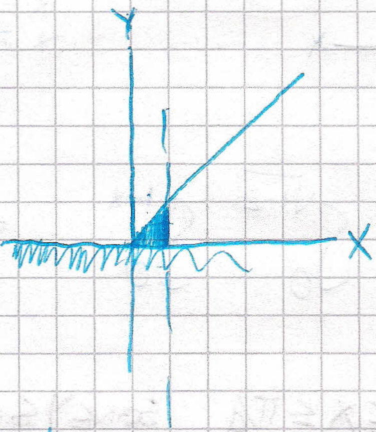
$y=0$

$y=x$

dxdy ?

dydx



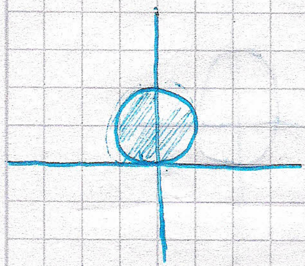


$$\int_0^1\int_0^y x^2 \quad dx\,dy$$

$$\int_0^1\int_0^x x^2 \quad dy\,dx$$

⑦ $\int_0^{\pi}\int_0^{2\,\text{sen}\theta} f(r\cos(\theta), r\,\text{sen}(\theta))\,dr\,d\theta$

Escriba la region D en cartesianos



$x^2+(y-1)^2=1$

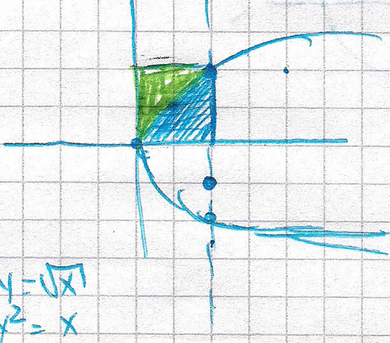
$x^2+(y-1)^2 \leq 1$

⑧ $\iint_D y^2 x e^{y^7}$

D = $0 \leq x \leq 1$

$\sqrt{x} \leq y \leq 1$

$\int_0^1 \int_{\sqrt{x}}^1 y^2 x e^{y^7}\,dy\,dx$



$\int_0^1 \int_0^{y^2} y^2 x e^{y^7}\,dx\,dy$

$y=\sqrt{x}$

$y^2=x$

$y=1$

$\int_0^1 y^2 e^{y^7} \frac{y^4}{2}\,dy$

$\frac{1}{2}\int_0^1 y^6 e^{y^7}\,dy$

$u=y^7$

$du=7y^6$

$\frac{du}{7}=y^6dy$

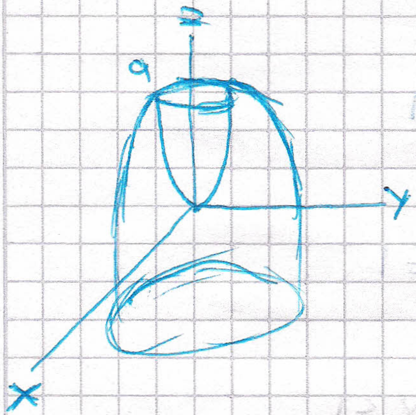
$\frac{1}{14}\int_0^1 e^u$

$\boxed{\frac{1}{14}(e-1)}$

⑨ $z = 8(x^2+y^2)$ $z = 9-x^2-y^2$

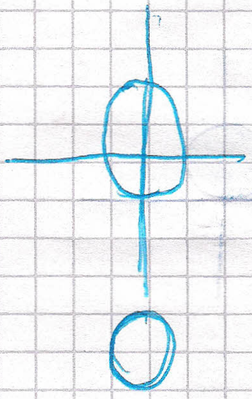
Dibuje y volumes

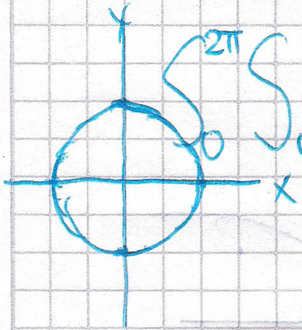
$z = 8(x^2+y^2)$ $z = 9-(x^2+y^2)$



$z = 8r^2$

$z = 9-r^2$





$$\int_0^{2\pi}\int_0^{1}\int_{8r^2}^{9-r^2} r\, dz\, dr\, d\theta$$

$8r^2 = 9-r^2$

$9r^2 = 9$

$r^2 = 9$

$r = 1$

$$\int_0^{2\pi}\int_0^{1} 9-r^2-8r^2\ (r)\, dr\, d\theta$$

$9-9r^2$

$$9\int_0^{2\pi}\int_0^{1} r-r^3\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{1^2}{22} - \frac{1}{4}\, d\theta$$

$=$

$=$

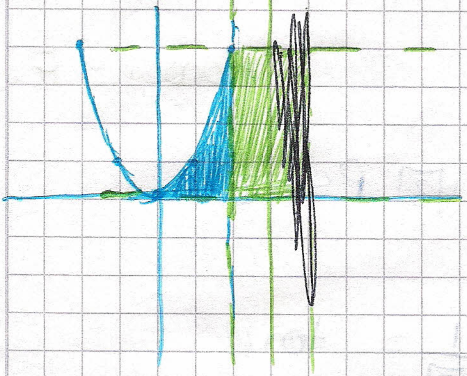
$\frac{1}{4}(2\pi)$

$\boxed{\frac{\pi}{2}}$

$\boxed{\frac{9\pi}{2}}$

⑩ $\int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx + \int_2^3 \int_0^4 f(x,y)\,dy\,dx$

Exprese como una sola integral



$y=x^2$

dydx?

dxdy

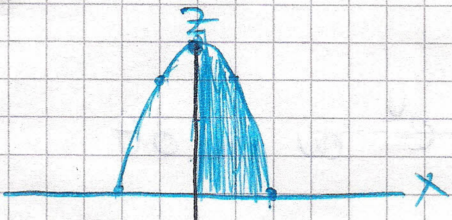
$\int_0^4 \int_{\sqrt{y}}^3 f(x,y)\,dx\,dy$

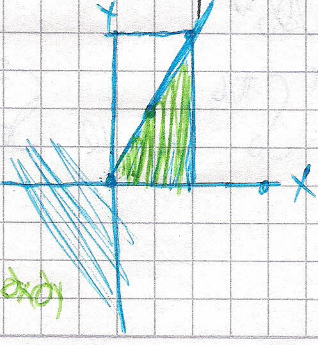
~~$\int \int f(x,y)\,dy\,dx$~~

⑪ Primer Octante Volumen

$z=4-x^2$
$y=0$
$z=0$
$y=2x$

dydxdz ?
dzdxdy ?





$\int_0^4 \int_0^{\sqrt{4-z}} \int_0^{2x} dy\,dx\,dz$

$\int_0^4 \int_0^{2y} \int_0^{4-x^2} dz\,dx\,dy$

ρ^2

⑫ Evalue

$$\int_R e^{(x^2+y^2+z^2)^{3/2}} \qquad \overset{\rho^2}{}$$

R= ~~•~~ $x^2+y^2+y^2=9$ = ρ

$z=\sqrt{3(x^2+y^2)}$ = Angulo

Plano = θ

$\rho^2=9$

$\rho=3$

$\rho\cos\alpha=\sqrt{3}\,\rho\,\text{sen}\,\alpha$

$\frac{1}{\sqrt{3}}=\tan\alpha$

$\pi/6$

$$\int_0^{2\pi}\int_0^{\pi/6}\int_0^{3} e^{\rho^3}\,\rho^2\,\text{sen}\,\alpha\,d\rho\,d\alpha\,d\theta$$

$u=\rho^3$

$du=3\rho^2$

$\frac{du}{3}=\rho^2\,d\rho$

$$\frac{1}{3}\int_0^{2\pi}\int_0^{\pi/6}\text{sen}\,\alpha\int_0^{27}e^{u}\,du\,d\alpha\,d\theta$$

$$\frac{e^{27}-1}{3}\int_0^{2\pi}\int_0^{\pi/6}\text{sen}\,\alpha\,d\alpha\,d\theta$$

DD MM AA

$$\frac{e^{27}-1}{3}\int_0^{2\pi} -\cos u \Big|_0^{\pi/6} \, d\theta$$

~~[illegible]~~

$$-\cos\left(\frac{\pi}{6}\right) + \cos(0)$$

Masa = ?

$$\left(\frac{e^{27}-1}{3}\right)\left(-\frac{\sqrt{3}}{2}+1\right)\left(2\pi\right)$$

⑬

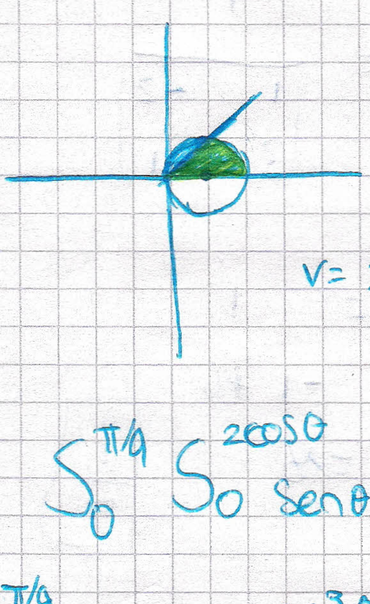
$(x-1)^2 + y^2 = 1$

$y=0$ $\quad$ $y=x$ $\quad$ $\sqrt{(x-x_0)^2 + (y-y_0)^2}$

D $\quad$ $\sqrt{(x-x)^2 + y^2}$

$\delta = y$

$y = r \operatorname{sen}\theta$



$r = 2\cos\theta$

$$\int_0^{\pi/4}\int_0^{2\cos\theta} \operatorname{sen}\theta \, r^2 \, dr \, d\theta$$

$u = \cos\theta$

$-du = \operatorname{sen}\theta$

$\frac{2}{3}$

$\frac{16}{16} - \frac{4}{16}$

$$\frac{8}{3}\int_0^{\pi/4} \operatorname{sen}\theta \cdot \cos^3\theta \, d\theta$$

$$\frac{8}{3}\int_{\sqrt{2}/2}^{1} u^3 \, du$$

$\frac{\cancel{12}^{6}}{\cancel{16}_{8}}$ $\quad$ $\frac{3}{4}$

$\frac{6}{12}$ $\quad$ $\boxed{\frac{1}{2}}$

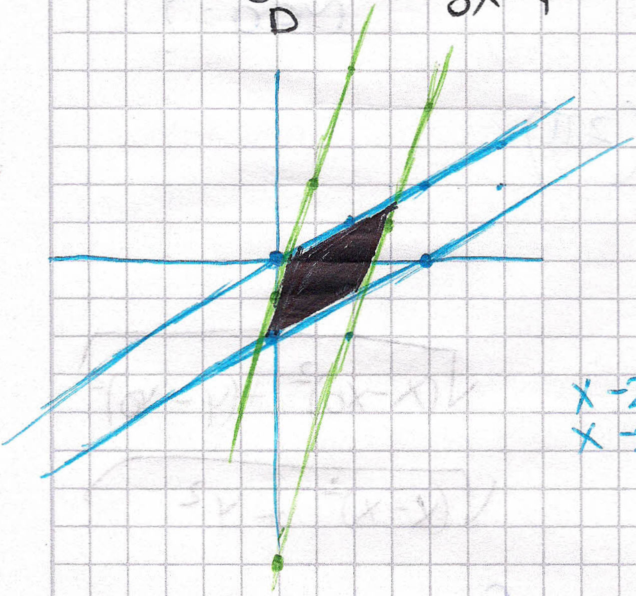
⑭

$x = 2y$
$x = 2y + 4$

$y = 3x - 1$
$y = 3x - 8$

Halle
la integra

$$\iint_D \frac{x-2y}{3x-y}\, dA$$



$u = x - 2y$

$v = 3x - y$

$x - 2y = 0$
$x - 2y = 4$

$3x - y = 1$
$3x - y = 8$

$$\frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix}}$$

$$= \frac{1}{-1 + 6}$$

$= \cancel{8}$

$$\boxed{\frac{1}{5}}$$

$$\int_1^8 \int_0^4 \frac{u}{v} \frac{1}{5}\, du\, dv$$

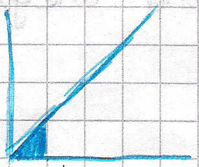
~~[illegible]~~

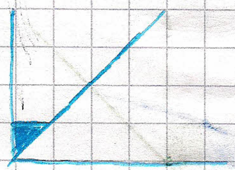
$$\frac{16}{2} \int_1^8 \frac{1}{v}\, dv$$

$$\boxed{8 \ln (8) / 5}$$

dx dy dz

⑮ $$\int_0^1\int_0^x f(x,y)\,dy\,dx = \int_0^1\int_0^y f(x,y)\,dx\,dy$$





FALSO

⑯ $$\int_{-1}^{1}\int_{-1}^{1}\cos(x^2+y^2)\,dxdy = 4\int_0^1\int_0^1\cos(x^2+y^2)\,dx\,dy$$

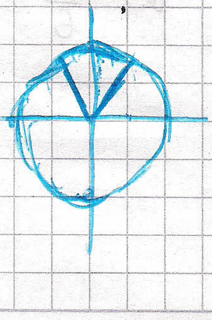
$\cos(x^2+y^2)$ es funcion par

VERDADERO

~~un volumen de una esfera~~ ~~[illegible]~~

⑰ Evalue

$$\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{x^2+y^2}}\frac{1}{\sqrt{x^2+y^2}}\,dz\,dy\,dx$$



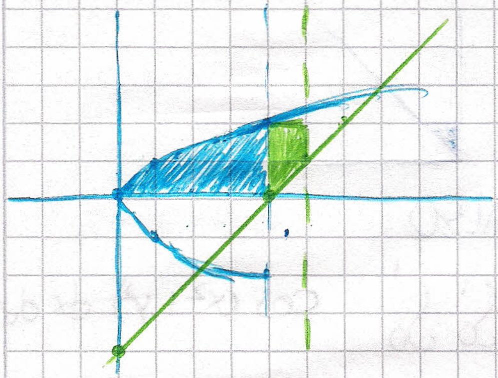
$$\int_0^{\frac{\pi}{2}}\int_0^2\int_0^r\frac{1}{r}\,r\,dz\,dr\,d\theta$$

$$\frac{\pi}{\cancel{2}}\cdot\cancel{2}\cdot\boxed{\pi}$$

$y^2+x^2=4$

⑱ $\int_0^4 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx + \int_4^5 \int_{x-4}^{2} f(x,y)\,dy\,dx$

una integral



$y^2 = x$

$y = x - 4$

[illegible]

NO SE PUEDE HACER EN UNA SOLA INTEGRAL

⑲ Sea $a>0$

$x^2+y^2+z^2 \le a^2$

Primer Octante $\pi/2$

$z = \sqrt{3x^2+3y^2}$ $\quad z = \sqrt{\frac{x^2}{3}+\frac{y^2}{3}}$

$\rho^2 = a^2$ $\quad \rho\cos\varphi = \sqrt{3}\,\rho\,\mathrm{sen}\,\varphi$ $\quad \varphi = \frac{\pi}{6}$

$\rho = a$

$\frac{1}{\sqrt{3}} = \tan\varphi$

$\le \varphi$

$z = \frac{1}{\sqrt{3}} \rho \operatorname{sen} \varphi$ $\qquad$ $\frac{\pi}{6} \leq \varphi \leq \frac{\pi}{3}$

$\rho \cos \varphi = \frac{1}{\sqrt{3}} \rho \operatorname{sen} \varphi$ $\qquad$ $0 \leq \rho \leq a$

$\sqrt{3} = \tan \varphi$ $\qquad$ $0 \leq \theta \leq \pi/2$ primer octante

$\frac{\pi}{3} = \varphi$

$$\int_0^{\pi/2} \int_{\pi/6}^{\pi/3} \int_0^{a} \rho^2 \operatorname{sen} \varphi \, d\rho \, d\varphi \, d\theta$$

[redacted]

$$\frac{1}{3} \int_0^{\pi/2} \int_{\pi/6}^{\pi/3} \operatorname{sen} \varphi \quad a^3 \quad d\varphi \, d\theta$$

$$\frac{a^3}{3} \int_0^{\pi/2} -\cos \varphi \Big|_{\pi/6}^{\pi/3} d\theta$$

$$-\frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$\frac{a^3(\sqrt{3}-1)}{3 \cdot 2} \quad \frac{\pi}{2}$$

$$\boxed{\frac{a^3 \pi (\sqrt{8}-1)}{12}}$$

⑳ Use la transformacion $x = \frac{(u+2v)}{3}$

$y = \frac{(u-v)}{3}$

Para evaluar la integral

$$\int_0^{2/3}\int_y^{2-2y} (x-y)^{1/2}(x+2y)^{1/2}\,dx\,dy$$

[redacted]

$x = \frac{u+2v}{3}$

$3x = u+2v$ $\qquad$ $3y = u-v$

$3x-2v = u$ $\qquad$ $3y+v = u$

$3x-2v = 3y+v$ $\qquad$ [redacted]

$3x-3y = 3v$ $\qquad$ $3y+x-y = u$

$\boxed{v = x-y}$ $\qquad$ $\boxed{2y+x = u}$

$y=0$

$u=v$

DD MM AA

$$\frac{1}{\begin{vmatrix} U_x & U_y \\ V_x & V_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{1}{-1-2} = \frac{1}{3}$$

Para U

* $X = Y$

* $2 - 2Y = = X$

$\textcircled{2} = 2Y + X$

~~[illegible]~~ $2 - 2X = X$

~~[illegible]~~ $X = 3/2$

~~$X = 3/2$~~

~~[illegible]~~

$\boxed{Y=0}$ $\boxed{Y=X}$

$\boxed{U=0}$

$0 \leq U \leq 2$

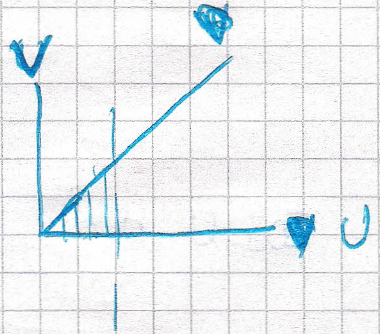
Para V

* $X = Y$

* $Y = 0$

$V = Y - Y$

$V = \textcircled{0}$



$0 \leq U \leq 2$

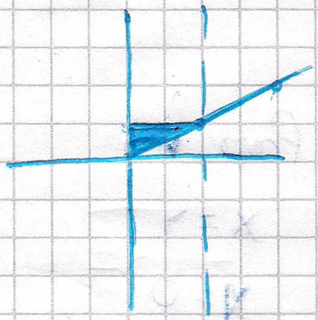
$0 \leq V \leq U$

$$\int_0^2 \int_0^U \sqrt{V} \sqrt{U} \frac{1}{3} dv\, du \quad \frac{1}{\ } \frac{2}{3} \int_0^2 U^{1/2} \quad U^{3/2}$$

$$U^2\, du$$

$$\int_0^2 \sqrt{U} \quad \frac{2V^{3/2}}{3} \Big|_0^U \frac{1}{3} du = \quad = \frac{2}{9} 8 \quad \frac{16}{9} \frac{1}{3} \quad \boxed{\frac{16}{27}}$$

㉑ $\int_0^2 \int_{x/2}^1 \cos(y^2)\,dy\,dx$



$y = \frac{x}{2}$

$$\int_0^1 \int_0^{2y} \cos(y^2)\,dx\;dy$$

$$\int_0^1 \cos(y^2)\;(2y)\;dy$$

$u = y^2$

$du = 2y\,dy$

$$\int_0^1 \cos u\;du$$

$$\text{Sen}(u)\Big|_0^1$$

$$\text{Sen}(1) - \cancel{\text{Sen}(0)} \to 0$$

$$\boxed{\text{Sen}(1)}$$

㉒ Primer octante

$2x + y + z = 5$

$x = 0$

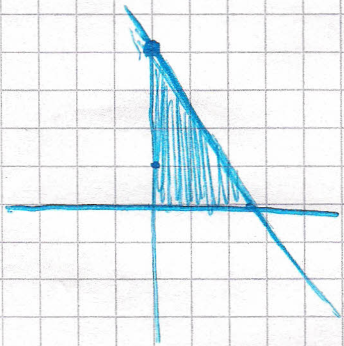
$y = 0$

$z = 0$

$dz\,dy\,dx$?

$z = 5 - 2x - y$

$$\boxed{\int_0^{5/2}\int_0^{5-2x}\int_0^{5-2x-y} dz\ dy\ dx}$$



$2x + y = 5$

$y = 5 - 2x$

㉓ Coordenadas esfericas
Primer octante

Dentro de la esfera
debajo del cono

$x^2 + y^2 + z^2 = 4$

$z = \sqrt{x^2 + y^2}$

$\rho\cos\varphi = \rho\,\mathrm{sen}\,\varphi$

$1 = \tan\varphi$

$\frac{\pi}{4}$

$\rho^2 = 4$

$\rho = 2$

~~$0 \le \varphi \le \pi/4$~~

$\pi/4 \le \varphi \le \pi/2$

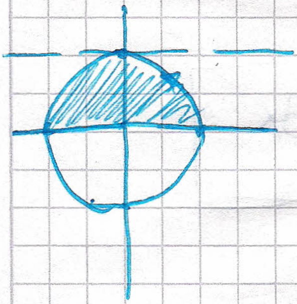
$0 \le \rho \le 2$

$0 \le \theta \le \pi/2$

$$\int_0^{\pi/2}\int_{\cancel{0}\,\pi/4}^{\cancel{\pi/4}\ \pi/2}\int_0^{2} \rho^2\,\mathrm{sen}\,\varphi\ d\rho\,d\varphi\,d\theta$$

㉔ $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dx\, dy$

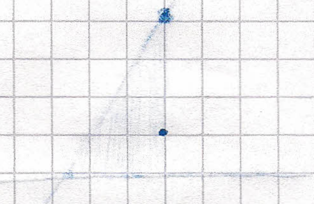
~~[illegible]~~



$x = \sqrt{4-y^2}$

$x^2+y^2=4$

$y=2$
$y=0$



$\int_0^{\pi} \int_0^{2} r\, dr\, d\theta$

$\frac{1}{\cancel{2}}(\pi)\, 4$

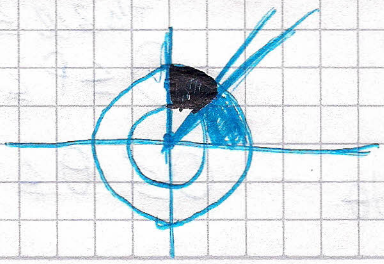
$\boxed{2\pi}$

㉕ Calcule

$\iint_D \frac{e^{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}\, dA$

Primer cuadrante

$D =$ $x^2+y^2=1$
$x^2+y^2=4$
$x=0$
$y=\sqrt{3}\,x$



$\sqrt{1} \le \sqrt{3} \le \sqrt{4}$

$1 \le \sqrt{3} \le 2$

~~$\tan 2\theta = \sqrt{3}$~~

$r \operatorname{sen}\theta = \sqrt{3}\, r\cos\theta$

~~sen~~ $\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$$\int_{\pi/3}^{\pi/3} \int_1^2 \frac{e^r}{\cancel{r}}\,\cancel{r}\,dr\,d\theta$$

$$= \int_{\pi/3}^{\pi/2} e^2 - e \quad d\theta$$

~~$\frac{\pi}{3}(e^2 - e)$~~

$\frac{\pi}{2} - \frac{\pi}{3}$ $\quad \frac{3\pi - 2\pi}{6}$

$\frac{\pi}{6}$

$$\boxed{\frac{\pi}{6}\left(e^2 - e\right)}$$

(26) $z = 2\sqrt{x^2+y^2}$

$z = 2$

$z = 2r$

$d = \sqrt{x^2+y^2}$
$\quad r$

$M = ?$

Cilindricas

$$\int_0^{2\pi} \int_0^1 \int_{2r}^{2} r\, r\, dz\, dr\, d\theta$$

~~$\int_0^{2\pi} \int_0^1 r^2 \quad 2 - 2r \, dr\, d\theta$~~

DD MM AA

$$\int_0^{2\pi}\int_0^1\int_{2r}^2 r^2\,dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 r^2\int_{2r}^2 dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 r^2(2-2r)\,dr\,d\theta$$

$$2\int_0^{2\pi}\int_0^1 r^2-r^3\,dr\,d\theta$$

$$2\int_0^{2\pi}\frac{r^3}{3}-\frac{r^4}{4}\Big|_0^1\,d\theta$$

$$\frac{1}{3}\quad\frac{1}{4}\qquad\frac{4-3}{12}$$

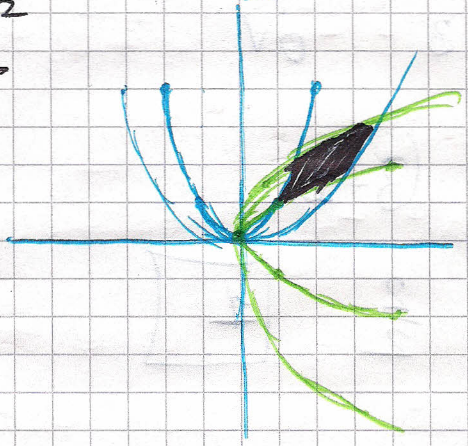
$$2\,\frac{1}{12}\qquad\frac{1}{6}\,2\pi\qquad\boxed{\frac{\pi}{8}}$$

27

$D = \quad x^2 = y \quad x^2 = 2y \quad y^2 = x \quad y^2 = 3x$

$y = \frac{x^2}{2} \qquad x = \frac{y^2}{3}$

$\iint_D \frac{x^2}{y}$



$\frac{x^2}{y} = 1 \qquad \frac{x^2}{y} = 2 \qquad \frac{y^2}{x} = 1 \qquad \frac{y^2}{x} = 3$

$u = \frac{x^2}{y}$

$v = \frac{y^2}{x}$

$$\frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \frac{}{\begin{matrix} \frac{2x}{y} & -\frac{x^2}{y^2} \\ -\frac{y^2}{x^2} & \frac{2y}{x} \end{matrix}}$$

$\boxed{\frac{1}{3}}$ Uacabrono $\frac{2\cancel{x}}{\cancel{y}} \cdot \frac{2\cancel{y}}{\cancel{x}} - \frac{\cancel{y^2}}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{\cancel{y^2}}$

$4 - 1$

$$\int_1^3 \int_1^2 u \quad \frac{1}{3} \, du \, dv$$

$$\frac{1}{3 \cdot 2} \int_1^2 3 \quad dv$$

$$\frac{1}{2} \quad (3-2)$$

$$\frac{2}{2} \quad \boxed{1}$$